IS A MOVING STAR RETARDED BY THE REACTION OF ITS OWN RADIATION?

By Leigh Page

Sloane Physical Laboratory, Yale University

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A question of some interest to the astronomer is whether or not a body in motion, such as a star, is retarded by the reaction of its own radiation. For, on the electromagnetic theory of radiation as developed by Maxwell and his followers, a beam of radiant energy is supposed to have a quasi-momentum, such that if a body emits energy in a single direction it will lose momentum and in consequence suffer a reaction tending to push it in the opposite direction. Now if a star is at rest, and in thermal equilibrium, it follows from symmetry that it will radiate equally in all directions, and there will be no resultant impulse. If, however, the star is in motion, classical electrodynamics leads to a greater emission in the forward direction than in the backward, and consequently it would appear at first sight as though there should be a retardation which would ultimately bring the star to rest. The problem has been treated in some detail by Professor Sir Joseph Larmor in the *Proceedings of the Fifth International Congress of Mathematicians*,[1] held at Cambridge in 1912, and he finds the resistance to motion due to the radiation to be

$$F = -vR/c^2$$

where v is the velocity of the star, R the rate of emission of energy, and c the velocity of light.

Apart from its intrinsic interest, Larmor's result is of importance in that it would constitute, if correct, a contradiction between classical electrodynamics and the Principle of Relativity (reference here is to the relativity of constant velocity systems, not to the broader conception of general relativity recently developed by Einstein). It is known, however, that the connection between classical electrodynamics and the Principle of Relativity is very close. Lorentz obtained the relativity transformations in his effort to give the electrodynamic equations for a moving system the same form as for a fixed system, even before Einstein advanced the relativity idea, and the author has shown that the electrodynamic equations can be derived in their entirety and exactly from the kinematical transformations of relativity and the assumption that each and every element of charge is a center of uniformly diverging tubes of strain.[2]

Now to calculate rigorously from the electrodynamic equations the reaction on so complicated a mass as a star would be hopelessly involved. Fortunately, however, the problem can be simplified to the extent of dealing with a single oscillator, i.e., a single vibrating electron, and yet we can obtain a result that will be a perfectly general test of Larmor's expression. For the latter gives the retarding force as a function of the rate of total radiation and the velocity of the radiating body, and of these quantities alone. Hence if the ether exerts a reaction on a group of moving oscillators, it will exert a similar reaction on a single oscillator; and conversely, if there is no reaction on a single vibrating electron due to its drift velocity, there can be none on a group of such vibrators.

A rigorous solution of the problem for this relatively simple case shows the existence of no retarding force. Larmor's result is found to be invalid because of a tacit assumption underlying his reasoning which was introduced substantially in the following manner. In order not to complicate matters by the introduction of terms in the inverse second power, the radiation reaction is calculated by applying the electrodynamic equations to the surface of a moving sphere with the electron as center, whose radius is large compared to that of the electron (though small compared to a millimeter). In this way only those terms in the expressions for the electric and magnetic fields which involve the inverse first power of the distance from the electron need be retained. But the result obtained really gives the force on the electron and the ether inside the moving sphere, not that on the electron alone. Now, if the motion of the electron were undamped, the field inside this moving sphere would remain unchanged, and consequently the force found by Larmor would be that actually exerted on the electron. But as the electron is radiating, its motion must be damped unless energy is supplied from some outside source, and in that case it must be shown that no impulse accompanies the transfer of energy to the electron—a matter of considerable difficulty to treat rigorously. It is far simpler to consider the case of an oscillator left to itself and allowed to radiate at the expense of the energy of its vibration. For this case it is found that the force exerted on the electronic vibrator by the ether inside the moving sphere mentioned above is exactly equal and opposite to that due to the ether outside. Moreover, from the point of exchange of momentum, the law of conservation of momentum demands that

Momentum lost by electron = Momentum gained by ether outside sphere − Momentum lost by ether inside sphere.

The terms on the right hand side (the second of which is overlooked by Larmor) annul one another. Therefore a single moving oscillator is not retarded by its radiation field, and as already noted we can generalize this result and conclude that a moving body of any size and complexity suffers no retardation as a result of its emission of radiant energy.

In the analytical reasoning leading to this conclusion it is found necessary to develop the complete dynamical equation of the Lorentz electron through terms of the fifth order, for the most general type of motion. Previous derivations of this equation have been confined to some special case, such as quasi-stationary motion in a straight line. The general equation is here published for the first time:

$$F_x = \frac{e^2 f_x}{6\pi ac^2 (1-\beta^2)^{3/2}} - \frac{e^2 \dot{\mathbf{f}} \cdot \boldsymbol{\beta} f_x}{2\pi c^4 (1-\beta^2)^3} \cdots - \frac{e^2 \dot{f}_x}{6\pi c^3 (1-\beta^2)^2} + \frac{e^2 \ddot{f}_x a}{9\pi c^4 (1-\beta^2)^{5/2}} - \frac{e^2 \dddot{f}_x a^2}{18\pi c^5 (1-\beta^2)^3} \cdots$$

$$F_y = \frac{e^2 f_y}{6\pi ac^2 (1-\beta^2)^{1/2}} - \frac{e^2 \mathbf{f} \cdot \boldsymbol{\beta} f_y}{2\pi c^4 (1-\beta^2)^2} \cdots - \frac{e^2 \dot{f}_y}{6\pi c^3 (1-\beta^2)} + \frac{e^2 \ddot{f}_y a}{9\pi c^4 (1-\beta^2)^{3/2}} - \frac{e^2 \dddot{f}_y a^2}{18\pi c^5 (1-\beta^2)^2} \cdots$$

where the X axis is taken in the direction of motion, the Y axis is in any direction perpendicular to the velocity, a stands for the radius of the electron, f is its acceleration, and $\beta = v/c$ where v is the velocity of the electron and c that of light. The charge e is expressed in Lorentz's unit. It may be remarked that the form of this equation, in so far as it involves β, is in exact accord with the Principle of Relativity.

The analytical reasoning involved in obtaining these results from the electrodynamic equations will be given in a paper which has been submitted for publication to the *Physical Review*.

[1] *Proc. Fifth Int. Cong. Math.*, **1**, 1912, (207).

[2] Relativity and the Ether, *Amer. J. Sci.*, *New Haven*, **38**, 1914, (169).

ON ELECTROMAGNETIC INDUCTION AND RELATIVE MOTION. II.

By S. J. Barnett

Department of Physics, Ohio State University

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1. If a cylindrical condenser is placed in a uniform magnetic field with lines of induction parallel to its axis, and is short-circuited and rotated, together with the short-circuiting wire, about this axis, it becomes charged to a potential difference equal to the motional electromotive force in the wire, or the rate at which the wire cuts magnetic flux. If, however, the condenser and wire are fixed and the agent producing the magnetic field rotates, the relative motion being the same as before, the condenser does not become charged, as was proved[1] by precise experiment in 1912. This is the first case in electromagnetic induction in which the observed effect does not depend entirely on the relativity of the motion.